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# Topics in linear dynamics

A linear dynamical system is given by a (continuous linear) operator on a Banach space or, more generally, on a Fréchet space. Well-known concepts in linear dynamics are that of a hypercyclic operator (which demands the existence of a dense orbit) and that of a chaotic operator (which demands, in addition, the existence of a dense set of periodic points). Apart from being interesting in its own right, the study of hypercyclicity blends nicely methods from functional analysis, operator theory and classical complex analysis. For introductions to the theory of linear dynamical systems we refer to [1] and [2]. The cours will be largely based on [2].

There will be 12 lectures of 90 minutes. Below we will give a rough outline of these lectures. Depending on the interests of the audience and the progress made there may be some last-minute modifications.

## Week 1.

**Lecture 1.** We will start by recalling the basic notions of the theory of (non-linear) dynamical systems: quasi-conjugate dynamical systems, dense orbits, topologically transitive maps, chaotic maps, weakly mixing maps, mixing maps, and the Birkhoff transitivity theorem.
**Lecture 2.** From this lecture on we will study linear dynamics, that is, the dynamics of (continuous linear) operators. Operators with a dense orbit are called hypercyclic. We will introduce three classical hypercyclic operators, and we will revisit (weakly) mixing and chaotic operators in the light of linearity.
**Lecture 3.** We will provide various useful criteria (i.e., sufficient conditions) for an operator to be hypercyclic. The main result in this context is the fact that an operator is weakly mixing if and only if it satisfies the so-called Hypercyclicity Criterion.
**Lecture 4.** We study here the set of hypercyclic vectors, i.e., the vectors with a dense orbit. We will prove, in particular, the Herrero-Bourdon theorem on the existence of dense subspaces of hypercyclic vectors. We will also show that there are no hypercyclic operators on finite-dimensional spaces. And we will show that linear dynamics can be as complicated as non-linear dynamics.
**Lectures 5 and 6.** In these lectures we will introduce and study various classes of hypercyclic operators. We consider, in particular, weighted backward shift operators on sequence spaces, infinite order differential operators on the space of entire functions, and composition operators on spaces of holomorphic functions. Depending on time we might also look at operators on Hardy spaces: adjoint multiplication operators and composition operators.

## Week 2.

**Lecture 7.** We first study the spectrum of hypercyclic operators. We obtain Kitai's theorem that each component of the spectrum intersects the unit circle. As an application we show that no compact operator on a Banach space can be hypercyclic.
**Lecture 8.** In this lecture we show that there are 'many' hypercyclic operators. First we show that any infinite-dimensional separable Banach space supports a hypercyclic operator; we then show that, in some sense, the set of hypercyclic operators is even dense in the space of all operators.

**Lectures 9 and 10.** By the Baire category theorem, a countable collection of hypercyclic operators on the same underlying space automatically has a common hypercyclic vector. This is no longer so for an uncountable collection. We derive a sufficient condition for the existence of common hypercyclic vectors in that case, and we illustrate the result by some examples.

**Lectures 11 and 12.** Motivated by ergodic theoretic considerations, F. Bayart and S. Grivaux have recently introduced a strengthening of hypercyclicity, that of a frequently hypercyclic operator. We will define that notion, give some examples, and derive some of its properties.

## References


[1] F. Bayart and É. Matheron, *Dynamics of linear operators*, Cambridge University Press, Cambridge, 2009.

[2] K.-G. Grosse-Erdmann and A. Peris-Manguillot, *Linear chaos*, Springer, London, 2011.