A Linearly Implicit-Explicit scheme for Layered Sedimentation

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Abstract

Initially homogeneous suspensions of small solid particles dispersed in a vicous fluid do not always sediment in a smooth continuous fashion as described, for instance, by the well-knwon "modes of sedimentation" arising from Kynch theory [5, 6, 3]. Instead, layers of different concentrations (staircasing) are often observed after settling has proceeded for some time. This effect is particularly well documented in the paper by Siano [7]. In experiments, this phenomenon is accentuated when a very dilute suspension has an initial concentration gradient [7], that is, when the concentration below the suspension-supernate interface gradually increases with depth. The equation that could possibly describe this phenomenon as was speculated by Siano is the Cahn-Hilliard (CH) Equation $\frac{\partial c}{\partial t} = M\Delta(\psi'(c) - \gamma\Delta c)$, which was proposed in [1] and [2] as a simple model for the process of phase separation of a binary alloy at a fixed temperature. However, this equation has symmetric steady state solutions, hence CH is not a suitable model to simulate staircasing.

To break the symmetry, in [4] Lowengrub and Truskinovsky proposed the coupled Navier-Stokes-Cahn-Hilliard (NSCH) system. By means of Linearly Implicit -Explicit schemes (LIMEX) to solve NSCH in one dimension, we will investigate the possibility of obtaining simulations that develop staircasing as observed by Siano.

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